Course "Physics and Mechanics of Random Media"

École des Mines de Paris
One-week course November 13-17, 2017
60 Bd Saint-Michel, Paris
http://cmm.ensmp.fr/ESPRM/

Keywords*: random structures, variability, simulations, homogenization, upscaling, elasticity, fracture statistics, reliability, computer aided design of materials*

Lecturers:

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Christian Lantuéjoul, Centre de Géosciences, École des Mines de Paris, Fontainebleau.
François Willot, Centre de Morphologie Mathématique, École des Mines de Paris, Fontainebleau
Bruno Figliuzzi, Centre de Morphologie Mathématique, École des Mines de Paris, Fontainebleau
Jesús Angulo, Centre de Morphologie Mathématique, École des Mines de Paris, Fontainebleau
Anne Françoise Gourgues, Centre des Matériaux P. M. Fourt, École des Mines de Paris, Évry

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Location*: École des Mines de Paris (60 Bd Saint-Michel, Paris)*
Participants*: 30 maximum*

Goal:

Many solid media and materials (composites, granular media, metals, biomaterials, porous media, soils, rocks, etc.) encountered in materials sciences, geophysics, environmental sciences, energetics, hydrogeology,... display microstructures and structures of several length scales, showing often a non- deterministic disorder. A better understanding and prediction of the resulting multiscale and random nature of materials' mesoscopic and/or macroscopic properties requires a modeling approach based on a combination of probabilistic concepts with methods of physics and mechanics. The course, which aims to provide an introduction to this subject, will be given in a self-contained series of lectures and training sessions on computers.

First, motivated by a review of advanced experimental techniques for the microstructure description, and by typical results involving fluctuations present in plasticity, damage, fracture, and flows phenomena in porous media, basic tools of applied probability and random processes

are recalled. Then, probabilistic tools for the description random media and models together with their simulation are introduced. At the second stage, physics and mechanics of random media are presented from the standpoint of approximate solutions of partial differential equations with random coefficients. For example, linear electrostatics problems in random media are studied by means of perturbation expansion of the random electric and displacement fields, while bounds on the effective permittivitty and of elastic moduli are derived from variational principles. This approach of homogenization, which can be applied to other physical properties like the composition of permeability, or of the thermal conductivity, is illustrated by third order bounds.

The third area of focus concerns the use of numerical techniques (in particular FFT-based computation), to provide an estimation of homogenized properties of random media from Monte Carlo type simulations. Bounds and numerical techniques are then extended to non linear behaviours, like the plasticity of polycrystals. Given the importance of reliability problems in a multitude of engineering applications, several fracture statistics models (brittle, ductile, fatigue) are worked out from a probabilistic approach.

Structure of the course: Five full days in a single week. Lectures (70%) and practical training on computers (30%).

Course contents

Day 1 (Monday November 13): Introduction and basic concepts:

9h-9h15 General introduction (F. Willot) **(Room L213)**

9h15- 10h30 Introduction to random media and homogenization : from images to physical properties (F. Willot) **(Room L213)**

10h30-11h Break

11h- 12h30 Introduction to applied probability and probabilistic models (B. Figliuzzi) **(Room L213)**

14h-15h Introduction to the simulation of random variables (C. Lantuejoul) **(Room L213)**

15h-15h30 Break

15h30-17h30 Morphological characterization of random sets and of random functions: size, repartition, connectivity (B. Figliuzzi) **(Room L213)**

Day 2 (Tuesday November 14): Models and simulation of random media

9h-10h Examples of models and simulation of point processes (C. Lantuejoul) **(Room L213)**

10h-10h30 Examples of models of random sets (Boolean models) (J. Angulo) **(Room L213)**

10h30-11h Break

11h-12h30 Examples of models of random sets (Boolean model) (J. Angulo) **(Room L213)**

14h-15h Gaussian random functions: properties and (conditional) simulations (C. Lantuejoul) **(Room L213)**

15h-18h Training session on morphological characterization of images and on simulations with the software Micromorph (F. Willot, B. Figliuzzi) **(Room L027)**

Day 3 (Wednesday November 15): Homogenization of random media (linear properties): bounds and numerical techniques

9h-10h Electrostatics of random media: perturbation expansion of the random electrical and displacement fields; estimation of the effective permittivity, statistical approach of the Representative Volume Element (F. Willot) **(Room L213)**
10h-10h40 Classical and Hashin-Shtrikman variational principles; derivation of bounds of effective properties (F. Willot) **(Room L213)**
10h40-11h Break
11h-11h45 Third order bounds of the dielectric permittivity and of the elastic moduli of some models of random media. Examples of optimal microstructures (F. Willot) **(Room L213)**
11h45-12h45 Training session on the calculation of bounds of linear properties of random media (F. Willot) **(Room L027)**

14h- 15h Numerical homogenization of random media, Representative Volume Element (F. Willot) **(Room L213)**
15h– 15h30 Break
15h30-17h Training session on heat conduction with Fast Fourier transform (FFT) computations (F. Willot) **(Room L027)**

Day 4 (Thursday November 16): Transport in random media. Fracture Statistics, numerical techniques

9h-10h30 Material variability of mechanical properties at different scales (A.-F. Gourgues-Lorenzon) **(Room L213)**
10h30-10h45 Break
10h45-12h15 Material variability of mechanical properties at different scales (A.-F. Gourgues-Lorenzon) **(Room L213)**

14h-15h45 Scales and physical properties in porous media (B. Noetinger) **(Room L213)**
15h45-16h Break
16h-17h Variational methods for non linear composites (Y.-P. Pellegrini) **(Room L213)**

Day 5 (Friday November 17): Homogenization of random media (nonlinear properties, resonances)

9h-10h30 Variational methods for non linear composites (Y.-P. Pellegrini) **(Room L213)**
10h30-10h45 Break
10h45-12h30 Variational methods for non linear composites (Y.-P. Pellegrini) **(Room L213)**

14h-16h Resonances in the properties of composite media (Y.-P. Pellegrini) **(Room L213)**
16h30-18h Training session on resonances (Y.-P. Pellegrini) **(Room L022)**
18h-18h15 Conclusion

Readings

M.J. Beran, Statistical Continuum Theories, John Wiley (1968).

J. Besson, G. Cailletaud, J.L. Chaboche, S. Forest, Mécanique non linéaire des matériaux, Hermes, (2001).

Th. Bretheau, M. Bornert, P. Gilormini (eds), Homogénéisation en mécanique des matériaux. Vol 1, 2, Hermes (2001). D. Jeulin, (ed.), Advances in Theory and Applications of Random Sets, World Scientific, (1997).
D. Jeulin, Random structure models for composite media and fracture statistics, in Advances in

Mathematical Modelling of Composite Materials, K.Z. Markov (ed.), World Scientific, 239-289, (1994).

D. Jeulin and M. Ostoja-Starzewski, (ed.), Mechanics of Random and Multiscale Microstructures, CISM Lecture Notes N° 430, Springer Verlag, (2001)

D. Jeulin, S. Forest (eds) Continuum Models and Discrete Systems CMDS11, Mines Paristech, les Presses, 2008.

C. Lantuéjoul, Geostatistical Simulation. Models and Algorithms, Springer Verlag (2002)

G. Matheron, Eléments pour une théorie des milieux poreux, Masson, (1967)

G. W. Milton, The Theory of Composites, Cambridge Univ. Press (2002)

P. Ponte Castañeda, P. Suquet. "Nonlinear composites." Advances in Applied Mechanics 34 (1998): 171-302.

B. Sudret, Uncertainty propagation and sensitivity analysis in mechanical models - Contributions to structural reliability and stochastic spectral methods. Habilitation à diriger des recherches, Université Blaise Pascal, Clermont-Ferrand (2007)

P. Suquet (ed) : Continuum Micromechanics. CISM Lecture Notes N° 377. Springer- Verlag. 1997.

J.R. Willis, Variational and related methods for the overall properties of composites, Adv. Appl. Mech. 21, 2-78, (1981).

Prerequisites: Basic knowledge in probability theory, physics and mechanics of solids.
Examination: The students prepare a written project from data processed during the training sessions. The project is submitted 3 weeks after the course