ABSTRACT

The watersheds of a function are geometric features which are very useful in picture segmentation. We briefly and intuitively define the notion of watersheds, and we state that this transformation can be performed starting from the skeleton of the function. This skeleton is a particular case of a general morphological transformation called Thinning.

Two examples of use of the watersheds are then given: the first one drawn from contour detection of defects in weld radiographs and the other one from the segmentation of bubbles on an electrophoresis gel.

Introduction

The aim of this paper is to present the notion of watershed of a function in a very practical way. For this purpose, we shall define and compute watersheds for digitized functions. The digitization of these functions is performed according to a hexagonal frame.

Watersheds of a function

Definition

Let f be a function. Its graph may be considered as a topographical surface, on which a number of features may be defined. Among these, the minima of the function, and its watersheds are of some interest.

Let us consider a drop of water on this topographical surface. The water streams down, reaches a minimum height where it stops. With each minimum, we associate the set of all the points of the graph from which the water may come. Such a set is called a catchment basin. Several catchment basins can overlap: Their common points form the watersheds of the function. (see Figure 1).

Figure 1: Remarkable features of a function: minima, catchment basins, watersheds.

This intuitive definition of a watershed must be refined: to do this, we need the following definitions: Let $\mathcal{G}$ be the sampling hexagonal frame of the function, and put:

$$X_i = \{ x \in \mathcal{G}, f(x) \leq i \}$$

$X_i$ is the set of all points of the frame such that $f(x)$ is less or equal to $i$. Let us consider two distinct points $x$ and $y$ embedded in $X_i$. We can define the geodesic distance between these two points as the length of the smallest arc, provided it is contained in $X_i$ and join $x$ to $y$. Let us now consider two sets $X$ and $Y$, $Y \subseteq X$. $Y$ is assumed to be the union of $n$ connected components $K_j$. We can define the zone of influence of a component $K_j$ as the set of all points of $X$ at a finite geodesic distance from $K_j$ (that is, at a finite distance from the nearest point of $K_j$) and closer to $K_j$ than to any other $K_i$ (Figure 2). The set of points of $X$ which do not belong to any zone of influence is called skeleton by zone of influence of $Y$ with respect to $X$ and denote $SIZ(Y;X)$ [2].
Let \( Z \) be the set of watersheds and \( Z_j \) the subset of \( Z \) of those points which are at height \( j \). Let us assume \( Z_j \) is known, for \( k \leq j-1 \), \( X_j \), being the threshold of function \( f \) at level \( j-1 \); 
\[ X_j - Z_j \] is then the set of points whose height is less than \( j \) and which belong to only one catchment basin. It is then clear that the points of \( X_j \) equally distant from two different connected components belong to the watersheds at level \( j \). Then:

\[ Z_j = SKIZ(X_{j-1} - Z_{j-1}; X_j) \]
and
\[ Z = \bigcup_j Z_j \]

This definition provides a method for building the watersheds, in an iterative process.

**Computation of watersheds**

There exists a very tight relationship between the transformation which provides the watersheds of a function and a morphological transformation of functions called Thinning [3].

Let us consider a function \( f \) at point \( x \), and an hexagon centered at this point. Let \( T_1 \) and \( T_2 \) be two disjoint sub-sets of the hexagon (\( T_1 \) and \( T_2 \) need not be a partition of the hexagon). We define the thinning of \( f \) by \( T = (T_1, T_2) \) as a function \( g \) defined at point \( x \) as follows:

\[
g(x) = \sup_{y \in T_2} f(y) \quad \text{iff} \quad \sup_{y \in T_2} [f(y)] < f(x) \leq \inf_{z \in T_1} [f(z)]
\]

\[ g(x) = f(x) \quad \text{if not} \]

This function is denoted by \( g = f \circ T \). It is possible to use many elements \( T \). In particular, we can take the successive rotations of one element. The transformation is called iterative thinning and denoted by:

\[ f \circ \tau\! T = (((f \circ \tau_1) \circ \tau_2) \ldots \circ \tau_6) \]

Then, this particular thinning is called skeleton of the function [1] (Figure 3).

**Figure 3**: a) grey-tone function b) skeleton of the grey-tone function

We can prove that the watersheds of a function are nothing other than the closed arcs of the skeleton [4]. In other words, if we eliminate the barbs of the skeleton, we obtain the watersheds, as shown in Figure 4.

**Figure 4**: Computation of watersheds by elimination of barbs in the skeleton.
This interesting property provides a powerful method of computation of the watersheds.

**Watersheds and picture segmentation**

The watersheds transformation is very effective in picture segmentation and in contour detection. Let us illustrate these transforms with two examples.

**Weld Radiographs**

Many weld radiographs show various defects, different in size and in contrast. In order to detect the contours of these defects, we can perform the watersheds transformation on the gradient function of the picture. In actual fact, the defects correspond to points with a low gradient, as is the case for the background (Figure 5).

![Figure 5](image)

(a) Weld radiographs with defects  
(b) Watersheds of the gradient function

**Electrophoresis gel**

In this case, the problem is to detect contours and to separate bubbles produced by proteins during their migration on a gel. Two watersheds have been performed: the first one on the initial picture, in order to segment the bubbles, and the second one on the gradient function to detect their contours. The first transformation is also used to eliminate over-segmentation which may occur during contour detection (Figure 6).

![Figure 6](image)

(a) Electrophoresis gel  
(b) Watersheds of the grey-tone function  
(c) Contour detection
References


