Morphological Interpolation and Color Images

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Abstract

This paper regards the application of interpolation tools offered by mathematical morphology to color images. The approach presented here uses the lexicographic ordering to compare values of the pixels. These comparisons are performed in a new, transformed color space - ***the comparative color space****, which is essential with respect to the visual importance of color channels for the human perception.*

By combining the morphological median and the proposed approach to treatment of color images, an efficient and fully automatic method of creation of color interpolation sequence has been obtained. It enables entirely automatic creation of morphing sequence for some types of images.

However, certain classes of images contain sensitive areas. In order to control the behavior of such areas, control points are introduced. Starting from the control points and two initial images, a ***warped morphological median*** *is created, which is then used instead of a simple morphological median to create a deformation sequence for the mentioned images with sensitive areas.*

Introduction

This paper regards the application of interpolation tools offered by mathematical morphology to color images. Up to now, morphological interpolation methods were restricted to sets and to graytone images only. In this paper, its area of application is extended to color images. Two methods of color interpolation sequence generation are proposed. The first one is a fully automatic process, and seems to be appropriate for image conversion, at least one of which is textured. The second one involves some control points, the position of which must be known beforehand.

The paper is divided into 7 sections. Section 1 is dedicated to a presentation of existing morphological methods of sets and image. Section 2 presents the proposed way of color image processing. Section 3 describes the assumed solution of the color image interpolation. Section 4 shows how morphological interpolation can be combined with warping techniques. Section 5 describes the way of creation the interpolation sequence. Finally, section 6 presents two examples. Concluding remarks are situated at the end of this paper, in its last section.

1. Morphological interpolation

Mathematical morphology offers some powerful tools for the image interpolation. The theory was initially developed for sets [1,2,7,8]. In the approach described in the current paper an important part of this theory has been applied - **morphological median**.

In order to explain this notion, we may start from that of *the influence zones* of sets. If P_1 P_2 …P_n are disjoint sets, then the influence zone of P_i is the locus of those points which are closer to P_i than to any other set. By an extension of language, when two sets only are involved, P and Q say, with *Q* included in P ($Q \subset P$), the influence zone of *Q* with respect to P^c is still called the influence zone of P inside Q [1,2] :

$$IZ_P(Q) = \left\{ x : d(x,Q) < d(x,P^C) \right\} \qquad (1)$$

Where *d(x, A)* stands for the geodesic distance [12] between point *x* and set A. The contour of the influence zone is a geodesic SKIZ (skeleton by influence zones) of *Q* in *P*. The influence zone defined by Eq.1 represents a **median set** between two sets, one included in another:

$$M(P,Q) = IZ_P(Q) \qquad (2)$$

It has been proved in [7,8] that the median set satisfies the following representation :

$$M(P,Q) = \bigcup_{\forall\lambda} \left\{ (Q \oplus \lambda B) \cap (P \Theta \lambda B) \right\} \qquad (3)$$

Where $\oplus \lambda B$ is a dilation of size λ and $\Theta \lambda B$ represents an erosion of size λ, both with the elementary structuring element *B* [5,6].

In the case of a partial inclusion of X inside Y - case of two sets with non-empty intersection ($X \cap Y \neq \varnothing$), median set of X and Y is introduced as the influence zone of $X \cap Y$ in $X \cup Y$:

$$M(X,Y) = IZ_{(X \cup Y)}(X \cap Y) \quad (4)$$

This equation describes a new set, placed between two initial sets (see Fig.1). The contour of it represents the SKIZ of $X \cap Y$ in $X \cup Y$. Then we draw from Eq.3 that :

$$M(X,Y) = \bigcup_{\forall A} \{ [(X \cap Y) \oplus \lambda B] \cap [(X \cup Y) \Theta \lambda B] \} \quad (5)$$

This equation constitutes a base for an efficient algorithm of median set creation [1,2].

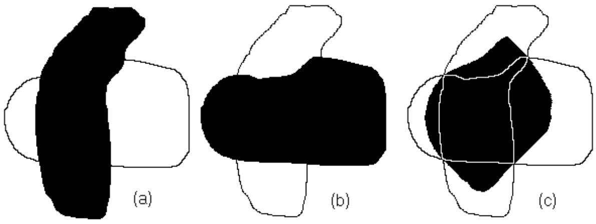

Fig. 1 Median set (c) of two sets (a) and (b)

One can observe that in Eq.5 the median element turns out to be an ***increasing mapping*** of its two operands. Hence it extends to a unique graytone mapping (where the transforms of the cross sections are the cross sections of the transforms). The implementation of this graytone operator is similar to that of the set case. We can formulate it in the following way:

$$m(f,g) = \sup \{ \forall \lambda : \inf [\delta^{\lambda}(\inf(f,g)), \varepsilon^{\lambda}(\sup(f,g))] \} \quad (6)$$

where $\lambda = 1,2,\ldots$ are increasing integer values, δ^{λ} and ε^{λ} are respectively dilation and erosion of size λ, performed with non-flat (cylindrical, cone, etc.) structuring element.

The last equation allows to construct the algorithm of median image generation. Starting from a pair *(f,g)* of initial image functions, we introduce the three working images z_0 w_0 m_0 as follows:

$$\begin{aligned} z_0 &= \inf(f,g) \\ w_0 &= \sup(f,g) \\ m_0 &= \inf(f,g) \end{aligned} \quad (7)$$

Iterated values are then computed using the following equations:

$$\begin{aligned} z_i &= \delta(z_{i-1}) \\ w_i &= \varepsilon(w_{i-1}) \\ m_i &= \sup[\inf(z_i, w_i), m_{i-1}] \end{aligned} \quad (8)$$

where δ and ε are respectively dilation and erosion with the elementary non-flat (cylindrical or cone) structuring element.

Iterations are performed until idempotence and finally:

$$m(f,g) = m_{\infty} = m_i \ , i : m_i = m_{i+1} \quad (9)$$

where *m(f,g)* is a new image - the morphological median of images *f* and *g*.

The median image is an important competitor of mean image, with some interesting advantages. An example of morphological median and the mean value of two images is shown on Fig.2 - median image is a totally new image enclosing the objects, shape of which is halfway between the shapes of the objects on the both initial images. Mean image contains in fact a mixture of two images containing both initial ones.

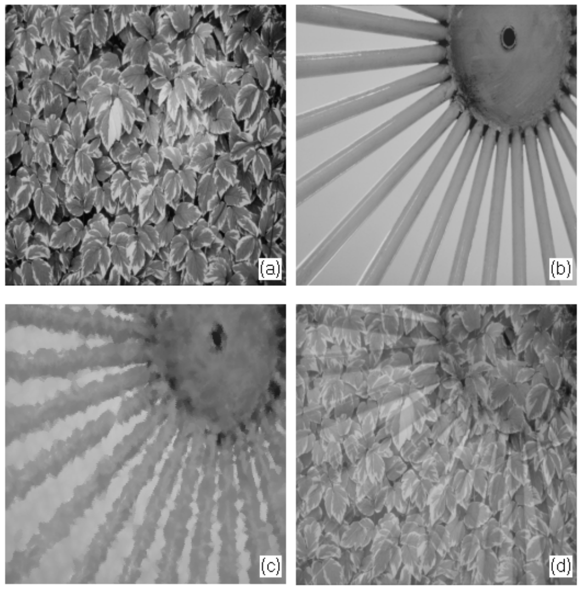

Fig. 2 Example of median image (c) and mean image (d) of two input images (a) and (b)

2. Lexicographic ordering and comparative color space

In every cartesian color space each pixel of an image is represented by a vector of color components. Consequently, one always has to compare triples of numbers when comparing pixels - e.g. in the morphological operators. One of the most popular ways of comparing is a lexicographic ordering [3,9]. It is based on the successive comparisons of vectors' elements and starts with the comparison of the vectors' components with the lowest indexes. It can be expressed by the following equations:

$$\begin{aligned} P < Q &\Leftrightarrow \exists i : 0 < i \le n ; \forall k : 0 < k < i , p_k = q_k \wedge p_i < q_i \\ P = Q &\Leftrightarrow \exists i : 0 < i \le n , p_i = q_i \\ P > Q &\Leftrightarrow \exists i : 0 < i \le n ; \forall k : 0 < k < i , p_k = q_k \wedge p_i > q_i \end{aligned} \quad (10)$$

where: $P = [p_1, p_2, \cdots, p_n]$ and $Q = [q_1, q_2, \cdots, q_n]$ are two vectors in n dimensional vector space.

This approach has however one important disadvantage - the *a priori* ordering of the importance of the vector components. In the case of the RGB color space, the ***r***-component is considered more important than the ***g***-component, while the ***b***-component is the less important one. But there is no reason to apply such an order of preference. To solve this problem a new color space is introduced, which is used only to perform a comparison of vectors [4]. This space is called a **comparative vector space** and the lexicographic ordering is applied there, instead of the initial color space. The initial color space (e.g. the RGB-one) is converted to the comparative one by using a 3x3 conversion matrix.

The general idea of conversion is based on the **visual importance** of color channels for the human perception. Fortunately in most real cases the components are not of the same importance for the human vision. It means that in case of points' comparison more important components can be compared before less important ones. In order to manage such a case one has to transform the initial (e.g. RGB) color space into the comparative one. The new space is used exclusively to compare the points from initial vector space and can be created by introducing linear combinations of the components.

Transformation of initial color space into comparative one is performed by using the following equation:

$$\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T = T_1 \cdot \begin{bmatrix} r & g & b \end{bmatrix}^T \qquad (11)$$

Where $V = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}^T$ is a vector in the comparative vector space and $\begin{bmatrix} r & g & b \end{bmatrix}^T$ is a vector in the RGB color space.

The coefficients of the transformation matrix can be chosen using different criteria.

The simplest one is based on the order of importance of the color channels for human perception. According to that the first compared value is the ***g***-value, second one is ***r***-value, and finally ***b***-value. It changes the order of comparisons in the lexicographical ordering. This change could be expressed by the following transformation matrix (***grb***-ordering):

$$T_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (12)$$

In order to obtain better convergence with human perception one can also consider linear combinations of vector elements, like eg. the luminance value. This value is considered as the most important factor during the comparison of vectors. In this case when comparing the points, one has to start with the comparison of the luminance values. In case of equality of both values compared, the visual importance of color channels is considered again. In such a case one has to check the ***g***-values (as the most important) of both vectors, and finally, if these values are also equal, one has to check the ***r***-values.

There are several ways of computing the luminance value. In our experiments two rules have been used.

The first one takes the length of a vector (norm Manhattan) as the luminance value.

Second rule is expressed by the following equation:

$$lum = 0.3r + 0.6g + 0.1b \qquad (13)$$

Appropriate transformation matrices are presented respectively below:

$$T_2 = \begin{bmatrix} 0.33 & 0.33 & 0.33 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad (14a)$$

$$T_3 = \begin{bmatrix} 0.3 & 0.6 & 0.1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad (14b)$$

As the experiments have shown, the last matrix produces the best visual results when used in morphological operators.

In the same manner different transformation matrices can also be generated.

The description presented above describes how two points in multidimensional color space are compared and consequently the *sup* and *inf* operators are constructed.

3. Color median image

The notion of median image for color images can be defined in the same way as in the case of graytone images. The definition presented in Eq.6 can be applied to both graytone and color images. The only difference lies in the *supremum* and *infinimum* operators. In the graytone case the comparison of numbers is performed. In the case of color images the comparisons which are necessary for *sup* and *inf* operators are executed on the vector values and must follow the rules of vector comparison from Eq.10 and one of the proposed transformation matrices (Eq.12,14). If one takes the above remarks into consideration, one may successfully apply the algorithm of median image generation previously introduced to color images.

4. Application of warping to the morphological interpolation

There exist certain classes of images which contain some sensitive areas. For example, when transforming one image of a human face into another, each of the interpolated frames should retain the important elements of the face (eyes, mouth, nose). Morphological median doesn't allow to point and specify particular objects in the image. In order to control the behavior of these sensitive areas control points must be applied. They point at the sensitive areas and ensure that these areas of both input images will not disappear within the interpolation sequence. The behavior of control points is controlled by a warping technique. The review of different warping methods is in [10,11,13]. In our experiments a bilinear mesh-warping [10] has been applied. Mesh-warping must be performed before the morphological median is computed. With the two already mentioned sets of control points and both images, a **warped morphological median** is produced. This new kind of median represents in fact "classic" morphological median. The difference is that input images for this median are not the two "pure" initial images, but warped initial images. The details are described later on.

The bilinear warping method is based on control points, placed in the nodes of the quadrilateral grid. Each of the control points has two positions: on the initial and on the final image. Warping algorithm converts an initial color (or graytone) image in such a way that the control points change their position from the initial to the final one. Value of all the other image points are recalculated using bilinear transformation.

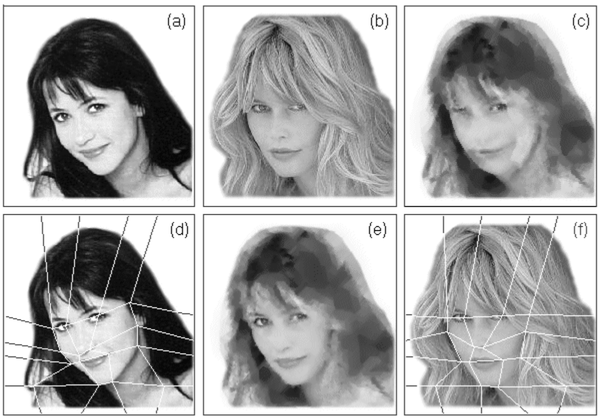

Fig. 3 Example of warped median. (a) and (b) - initial images,(c) - median image of (a) and (b), (d) and (f) initial images with grids of control points, (e) warped morphological median

The proposed warping method is based on recalculation of coordinates. For each point of final quadrilateral the coordinates of appropriate point from initial one are calculated. The value of the point from initial quadrilateral is copied into the final image. When the final quadrilateral is larger than the initial one, the content must be enlarged. The enlargement can cause problems which are typical for this kind of operation, like e.g. blocky appearance of enlarged parts of the image. They arise due to the necessity of copying a value of a pixel from initial image into more than one pixel on the final image - result of the conversion of real coordinates into integer ones. In order to solve this problem a bilinear interpolation has been applied. The value of a pixel at the final image is bilinearly interpolated using the values of four closest, known pixels on the initial image and the distances to these pixels.

The application of bilinear interpolation into bilinear warping solves the problem of blocky appearance on the enlarged areas and improves linearity and smoothness of contours.

Warping method, described above, is applied to improve the result of morphological median. Two initial images are warped in such a way that their control points are shifted to the same positions in both images. Later median image of two warped initial ones is computed. It can be described by the following algorithm:

I,J are the initial images, n is the number of control points, $C_I=\{p_1, .. , p_n\}$, $C_J=\{q_1,, .., q_n\}$ are the sets of control points on I and J

1. *Create new set $C_K=\{r_1, .. ,r_n\}$ containing mean positions of control points from C_I and C_J : for each i=1, .., n $r_i=0.5(p_i + q_i)$*
2. *Warp I from C_I to C_K (result: I')*
3. *Warp J from C_J to C_K (result: J')*
4. *Calculate morphological median of I' and J'*

An example is shown on Fig.5. It shows two grids of control points (d) and (f) - the control points themselves are nodes of these grids. 36 control points have been chosen, but only 14 represent "real" control points i.e. control points which have different positions on both images. The rest of the points are chosen only to supplement the grid in order to have a complete coverage of the image surface by the quadrilaterals. Two medians are shown: "classic" median image - (c) and warped median image - (f). On median image some of the important elements of human face have disappeared or have been deformed. The warped median, thanks to the possibility to control the important points on the image, retains all the important elements of the face.

5. Interpolation sequence

Color median is applied as a tool for the creation of the interpolation sequence. Two variants are distinguished: generation of **color interpolation sequence** and generation of **warped color interpolation sequence**.

In the first case the algorithm begins by calculating color median image of two initial images - intermediary image. Then, next two new medians are created. First one is a median of first initial image and intermediary image, the second one is a median of intermediary image and the second initial image. In the same way in each forthcoming iteration new color medians are generated. The general rule is that new median is obtained from two already generated ones. The output consists of several images - frames of interpolation sequence, which transform one initial image in order to obtain the another one. Interpolation sequence obtained is such a way contains a real change of objects' shapes (contours) instead of a sort of blending produced by cross-dissolving (see Fig.4).

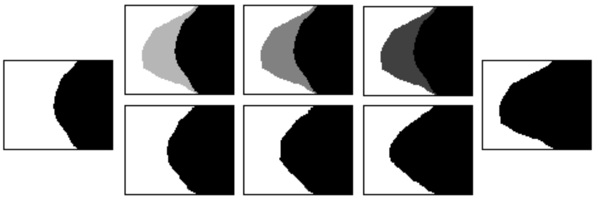

Fig. 4 Transformation of contour by cross-dissolving (top) and by morphological interpolation (bottom).

The sequence of warped color median is generated in a similar way. In this case not only the initial images make the input, but also do the two sets of positions of control points on both input images. The appropriate procedure of the generation of warped median creates two objects: warped median image and a new set of control points on the warped median image. This set contains intermediary mean positions of points. All the other rules for the sequence generation are similar to those described in previous paragraph.

The said number of images in a sequence must be equal to the power of 2 – this could be considered as a disadvantage. But in fact new medians don't have to be generated between each two already generated medians. This depends on the necessity of creation of new median. If two neighboring medians are close enough one to another there is no need to create new image between them. On the other hand, when the difference between two neighboring images is high there is no objection to generate one or more intermediary images between. Everything depends on the visual quality of the sequence.

6. Results

To illustrate the proposed interpolation methods, two examples are given. First example refers to color median image and color interpolation sequence (without warping). The interpolation sequence obtained by iterative color median image generations is presented on Fig.5. Color median was obtained according to Eq.6. Color pixels' comparisons were performed in the comparative vector space defined by Eq.14b. Result contains very good transformation of the image content from the first to the second image.

Second example is presented of Fig.6. It is a warped color interpolation sequence and contains the transformation of one human face into another. Face of one beautiful and well-known women is converted into the face of another one. This is a morphing sequence created using successive generations of the color warped morphological medians. Quadrilateral grids of control points used in this example is presented on Fig.3.

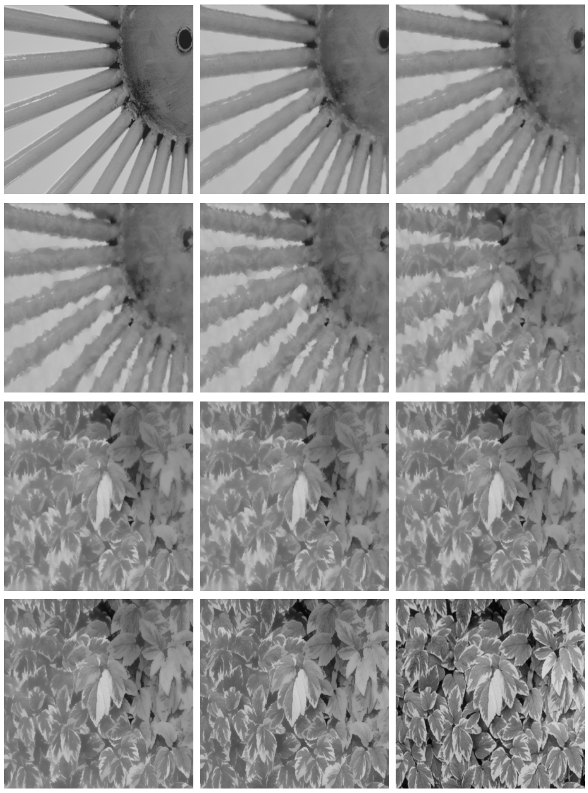

Fig. 5. Example of interpolation sequence generated using color morphological median

Fig.6. Example of morphing sequence generated using warped morphological median

Unfortunately, because of the publisher’s limitations figures 5 and 6 contain graytone images. Real images are, of course, in color.

Conclusion

In this paper the approach to color image processing has been proposed. It has some important advantages:

- it is relatively fast (space transformation and lexicographic ordering are fast),
- every Cartesian color space can be transformed to the comparative space,
- the morphological operations don’t need to be re-defined – they are exactly the same as in the graytone case.

Also two interpolation methods are presented in this paper. Both can be applied to the generation of morphing sequences between two color images. The first method is fully-automatic and gives very good results for such images, of which at least one is textured. The interpolation sequence contains a real change in the shape of objects on it, not only the simple mixing of both images like the linear methods of image interpolation produce. This kind of morphing has, in comparison with "classic" morphing, one great advantage - it is a **completely automatic method**. It doesn't need any control point, and because of that it requires no human assistance. But on the other hand, this way of generating morphing sequences yields correct results only for certain types of images. It produces interesting results when at least one of the initial images is textured and there is no need to transform some objects into another ones.

The second method - a combination of bilinear warping and color morphological median - allows to select the most important areas on the image, and to control the transformation process. For some image types, such an approach produces much better results than a sequence generated without warping. Especially when particular objects in the first image should be transformed into another objects in the second one. It also provides an alternative to traditional morphing techniques. Using the proposed method, the generation of a morphing sequence is much easier. This is due to the fact that the **number of control** points which have to be selected on the image **is considerably lower**. Sometimes just a few of them are sufficient to yield a correct morphing sequence, instead of the large set of contour points required by the traditional morphing methods. In the traditional morphing techniques with cross-dissolving control points must follow the shape of important contours. If contours are more complicated - the number of control points grows dramatically. It is due to the fact that in most of the morphing algorithms the cross-dissolving is performed. The linear operation cannot change shapes on the image - it can only mix two images. Morphological median can change the contour, and therefore in the morphing operation it is enough for the control points to indicate only the most important areas on the image. The change of shape between control points is performed without additional control points, the morphological median does not need them (see Fig.4). Moreover, when applied together with the automatic generation of control points, the proposed approach can also perform a fully-automatic morphing of images which include some sensitive areas.

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